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Unbiased Weibull capabilities indices using multiple linear regression

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Although the recently proposed Weibull process capability indices (PCIs) actually measure the times that the standard deviation (σ_x) is within the tolerance specifications, because they not accurately estimate neither the log-mean (μ_x) nor the σ_x values, then the actual PCIs are biased. This actually because μ_x and σ_x are both estimated without considering the effect that the sample size (n) has over their values. Hence, μ_x is subestimated and σ_x is overestimated. As a response to this issue, in this paper, μ_x and σ_x are estimated in function of n. In particular, the PCIs' efficiency is based on the following facts: (1) the derived n value is unique and it completely determines η, (2) the μ_x value completely determines the η value, and (3) the σ_x value completely determines the β value. Thus, now, since μ_x and σ_x are in function of n and they completely determine β and η, then the proposed PCIs are unbiased, and they completely represent the analyzed process also. Finally, a step by step numerical application is given.

KEYWORDS
multiple linear regression, process capability indices, sample size, Weibull distribution

1 | INTRODUCTION

The process capability indices (PCIs) were first formulated for normal behavior.[1] However, because any capable controlled process is subjected to noise factors and products are more complex and multifunctional, then the process behavior generally is not normal.[2,3] And in particular for the Weibull distribution, several methods to estimate the Weibull PCIs have been proposed. Among others, see Chen and Pearn,[4] Tang and Than,[5] Chang et al,[6] Vännman,[7] Albing,[8] Shauly,[9] and Piña et al.[10] Unfortunately, because none of them "addressed the direct relationships" between the Weibull β and η parameters with the log-mean μ_x and the log-standard deviation σ_x process parameters, then, they estimated μ_x and σ_x by using different approaches. Among those approaches, the most accurate is the one based on the Weibull/Gumbel and Gumbel/lognormal relationships given in Piña et al.[10] Regardless of that, because this approach did not consider the effect that the sample size n has over μ_x and σ_x, then it always biases the estimated PCIs. In particular, the bias occurs, (1) because the Gumbel/lognormal relationship given in Equation 9 Section 2.1, always subestimates μ_x. The subestimation occurs because in Equation 9, the Euler constant (γ) is higher than the corresponding mean (μ_y) of the response vector Y; and (2) because the Gumbel/lognormal relationship given in Equation 10 Section 2.1, always overestimates σ_x. The overestimation occurs because in Equation 10, $\sqrt{6}/\pi$ is higher than the corresponding standard deviation (σ_y) of the response vector Y.

On the other hand, in the proposed method, the bias of the estimated PCIs is completely eliminated because under multiple linear regression, the relationships between β and η with μ_x and σ_x are unique, and completely determined (see Section 4, Equations 28 and 30. And because the sample size n used to estimate μ_x and σ_x is also unique and it completely determines η. Thus, the proposed Weibull PCIs, by using μ_x and σ_x, also measure the times that σ_x is within the specifications limits. And since μ_x and σ_x are both estimated directly from the Weibull lifetime data, then the method by itself corrects the bias, and it completely represents the analyzed process.

The paper structure is as follows: Section 2 gives the generalities of the Weibull PCIs. Section 3 shows how to estimate the sample size n. Section 4 presents the Weibull and process relationships. In Section 5, the proposed method is given. The paper ends in Section 6 with the conclusions.

2 | GENERALITIES OF THE WEIBULL PCIs

The PCIs are referred as the inherent ability of a process to produce homogeneous parts for a sustained period under the given quality conditions. In particular, the PCIs measure the potential process performance as the times that the process standard deviation (σ) is within the specification limits. Thus, because σ depends on both the sample size (n) and on the mean (μ), then since μ also depends on n, then having an accurate estimation of n is critical to formulate the PCIs. Seeing that, observe that from a sample n data, σ is estimated as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}}. \tag{1}$$

In addition, observe from Equation 2 that the parameter μ used in Equation 1 also depends on n

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i. \tag{2}$$

Thus, since both Equations 1 and 2 depend on n, then the related PCIs also depend on n. On the other hand, the μ and σ parameters defined in Equations 1 and 2 are the mean and the standard deviation of the normal distribution. Thus, they are used in the normal PCIs. The most used normal PCIs are the process capability (C_p) index given by Juran[11] and the process ability (C_{pk}) index given by Kane,[12] with C_{pk} = min(C_{pu}, C_{pl}), where USL is the upper specification limit and LSL is the lower specification limit. Thus, the normal C_p index is given by

$$C_p = \frac{(USL-LSL)}{6\sigma}, \tag{3}$$

and the normal C_{pk} index is given by

$$C_{pk} = \min\left(C_{pu}, C_{pl}\right) = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right). \tag{4}$$

On the other hand, although as in the normal case, the Weibull PCIs given in Piña et al[10] where formulated to measure the times that the log-standard deviation σ_x is within the specification limits, because the used log-mean μ_x and σ_x values where both estimated without considering the effect that n has on μ_x and σ_x (see Equations 1 and 2, then they are inefficient. However, since these PCIs are the base of the proposed process, then in the next section the Weibull/Gumbel/lognormal relationships on which the PCIs where formulated are given.

2.1 | Weibull/Gumbel/lognormal PCIs

This section is mainly based on Piña et al[10]; thus, only the needed formulas are given. The PCIs were formulated on the basis of the relationship between the Weibull/Gumbel parameters (Crowder et al[13] Section 2.4 pp 17) given by

$$\mu_{EV} = \ln(\eta), \tag{5}$$

$$\sigma_{EV} = \frac{1}{\beta}. \tag{6}$$

And on the relationships between the Gumbel/lognormal parameters given by 1 study[14] Section 1.3.6.6.16.

$$\mu_{EV} = E(x) = \mu_x + \gamma\sigma_{EV}, \tag{7}$$

$$\sigma_{EV} = \frac{\sqrt{6}}{\pi}\sigma_x. \tag{8}$$

In Equations 5 and 6, μ_{EV} and σ_{EV} are the mean and the standard deviation of the Gumbel (minimum extreme type-I value) distribution. And in Equations 7 and 8, μ_x and σ_x are the mean and the standard deviation of the logarithm of the failure times estimated as in Equations 1 and 2. Finally, from Equations 7 and 8 (McCool,[15] Section 6.7), μ_x and σ_x are given by

$$\mu_x = \mu_{EV} - \gamma\sigma_{EV}, \tag{9}$$

$$\sigma_x = \frac{\pi}{\sqrt{6}}\sigma_{EV}. \tag{10}$$

In Equation 9, γ is the Euler constant ($\gamma = 0.577216\ldots$), and in Equation 10, $\pi/\sqrt{6} = 1.28255$. Finally, based on Equations 6 and 10, the Weibull capability index Cp defined in Equation 3, was formulated as

$$C_{pW} = \frac{[\ln(USL) - \ln(LSL)]}{\sqrt{6}\pi\sigma_{EV}} = \frac{\beta[\ln(USL) - \ln(LSL)]}{\sqrt{6}\pi}. \tag{11}$$

And based on Equations 5, 9, and 10, the C_{pk} index, defined in Equation 4 was formulated as

$$C_{pkW} = \min\left(C_{pu}, C_{pl}\right) = \min \left(\frac{\beta\sqrt{6}[\ln(USL) - \widehat{\mu}_x]}{3\pi}, \frac{\beta\sqrt{6}[\widehat{\mu}_x - \ln(LSL)]}{3\pi}\right). \tag{12}$$

Unfortunately, since Equations 7 to 10 do not consider the effect that n has over μ_x and σ_x, then Equations 11 and

12 biased their estimations. Before to estimate their biased, we need to determine first the "correct sample size" n to formulate the proposed unbiased Weibull PCIs.

3 | WEIBULL SAMPLE SIZE n

To formulate n, first, let mention that because the Weibull distribution is a time-dependent distribution, then the desired "lower reliability" index $R(t)$ for a desired time t is the critical variable to be measured. The reliability function, derived from the Weibull distribution[16] given by

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{13}$$

is given by

$$R(t) = \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\}. \tag{14}$$

On the other hand, from Equation 14, by knowing 3 of the 4 variables, the last one is completely determined. Thus, because in Weibull analysis, the lower $R(t)$ and t values are the customer requirements, and β is selected from engineering knowledge or historical data, then generally η has to be determined. For instance, suppose, the customer requirements are $R(t) = 0.96$, $t = 1500$ hours, and from line base product (similar products), $\beta = 3$, thus by using these values in Equation 14, the corresponding η value is estimated as

$$\eta = \frac{1}{-\ln[R(t)]^{1/\beta}}t. \tag{15}$$

From the above data in Equation 15, $\eta = 4356.388$ hours. As a consequence, the Weibull family, which the PCIs shall represent is W(3, 4356.388). Therefore, since in Equation 14, only η is unknown, then it should also be possible to estimate η on the basis of n. Fortunately, from Equation 15 and Piña et al,[17] the n value that holds with the given $R(t)$, t, and β values is given by

$$n = \frac{1}{-\ln[R(t)]}. \tag{16}$$

Here, it is important to highlight that Equation 16 depends only on $R(t)$, and although Equation 16 does not depend on t, β, or η, then once t and β have been already selected, Equation 16 completely determines η. Thus, n is robust under any desired (or used) t, β, or η values. Moreover, since n only depends on $R(t)$, then n is unique for the desired $R(t)$ index, and as a consequence, n also completely determines μ_x and σ_x. Finally, from Equation 15, η in terms of n is estimated as

$$\eta = n^{1/\beta}t. \tag{17}$$

Numerically, by using the above data in Equation 16, and then Equations 16 into 17, η is estimated as η = (24.496)^(1/3)(1500) = 4356.388 hours. Thus, now we have the correct n to estimate μ_x and σ_x, but to show how to use it to estimate μ_x and σ_x, let first show the Weibull and process parameters relationships.

4 | WEIBULL AND PROCESS PARAMETERS RELATIONSHIPS

The Weibull and process parameters relationships are based on the cumulative Weibull function given by

$$F(t) = 1 - \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{18}$$

which in linear form is given by

$$\begin{aligned} Y_i &= \ln\{-\ln[1-F(t_i)]\} = -\beta\ln(\eta) + \beta\ln(t_i) \\ &= b_0 + \beta X_i, \end{aligned} \tag{19}$$

where $F(t_i)$ is estimated by the Benard approximation[18,19] given by

$$F(t_i) = \frac{i-0.3}{n+0.4}. \tag{20}$$

From Equation 19, β is directly given by its slope, and η is given by

$$\eta = \exp\left\{\frac{-b_0}{\beta}\right\}. \tag{21}$$

On the other hand, the parameters of Equation 19, by using multiple linear regression, are given by

$$b_0 = \overline{Y} - \beta\overline{X}, \tag{22}$$

$$\beta = \frac{\sum_{i=1}^{n} Y_i\left(X_i - \overline{X}\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} = \frac{Sxy}{Sxx}. \tag{23}$$

And the related coefficient of multiple correlation (R^2) is

$$R^2 = \frac{\beta\sum_{i=1}^{n} Y_i\left(X_i - \overline{X}\right)}{\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2} = \frac{\beta Sxy}{Syy}. \tag{24}$$

Thus, because β in Equation 24 is given by

$$\beta = \frac{R^2 Syy}{Sxy}. \tag{25}$$

Then, by taking from Equation 23 $Sxy = \beta Sxx$ and by replacing it in Equation 25, β is directly related to σ_y, σ_x, and R, by

$$\beta = \frac{R\sigma_y}{\sigma_x}, \tag{26}$$

where σ_x is the standard deviation of the logarithm of the lifetime data, which can be estimated by using Equation 1. And σ_y is the standard deviation of the median rank estimation defined in Equation 20. From a set of data, σ_y is estimated as

$$\sigma_y = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \mu_y\right)^2}{n-1}}. \tag{27}$$

On the other hand, from Equation 20, we observe that σ_y is also in function of n defined in Equation 16. However, because Equation 16 is unique and it depends only on *R(t)*, then σ_y is constant in the analysis. In addition, observe from Equation 26 that for $R = 1$, the expected σ_x is given by

$$\sigma_x = \frac{\sigma_y}{\beta}. \tag{28}$$

Thus, by using Equation 22 into Equation 21, η is given by

$$\eta = \exp\left(\mu_x - \frac{\mu_y}{\beta}\right). \tag{29}$$

Finally, from Equation 29, μ_x is given by

$$\mu_x = \ln(\eta) + \frac{\mu_y}{\beta}. \tag{30}$$

Summarizing*:* By comparing Equation 9 from Section 2.1 with Equation 30, we observe that in Equation 9, γ should represent the mean μ_y, but because γ is constant and μ_y depends on n, then Equation 9 is inefficient to estimate μ_x. Similarly, by comparing Equation 10 from Section 2.1 with Equation 28, we observe that in Equation 10, $\sqrt{6}/\pi$ should represent σ_y, but because $\sqrt{6}/\pi$ is constant and σ_y depends on n, then Equation 10 is also inefficient to estimate σ_x. Finally, observe that μ_x and σ_x in terms of n are estimated as in Equations 1 and 2. Thus, since now we have μ_x and σ_x in terms of n, then the proposed PCIs are as follows.

5 | PROPOSED METHOD

Since the proposed PCIs are based on the logarithm of the lifetime data, which is asymptotically normal,[20,21] then they are analogous to those of the normal distribution defined in Equations 3 and 4. The only difference is that the Weibull PCIs are measured in logarithm scale. The proposed PCIs are

$$C_{pW} = \frac{(USL - LSL)}{6\sigma_x}, \tag{31}$$

$$C_{pkW} = \min\left(C_{pu}, C_{pl}\right) = \min\left(\frac{USL - \mu_x}{3\sigma_x}, \frac{\mu_x - LSL}{3\sigma_x}\right). \tag{32}$$

In Equations 31 and 32, μ_x and σ_x are estimated as in Equations 28 and 30. And from a set of data they are estimated as in Equations 1 and 2. On the other hand, the *USL* and the *LSL* in Equations 31 and 32 are the logarithm of the maximum and minimum expected times of the related Weibull family of the analyzed process (see steps 4 and 5 of Section 5.1). Finally, when in the Weibull analysis, the time is the variable of interest, because "the higher the time the better," then in Equation 32, only the C_{pl} is used. On the other hand, when the time is not the variable of interest, then from Equation 32 can be selected as the C_{pk} index, the min(C_{pu},C_{pl}) .The steps to implement the proposed PCIs are as follows.

5.1 | Steps of the proposed method

Step 1 Determine the β value, and the *R(t)* and *t* requirements. Then by using those values in Equation 15, determine the corresponding η parameter.

Step 2 By using the β and η parameters of step 1, form the corresponding Weibull family.

Step 3 By using the *R(t)* value of step 1 into Equation 16, estimate n. Then by using n into Equation 20 and then Equation 20 into Equation 19, estimate the corresponding Y vector. And by using the Y elements in Equations 2 and 27, estimate the corresponding μ_y and σ_y values.

Step 4 By using the β and η parameter of step 1 and the Y_i elements of step 3 into Equation 19, estimate the corresponding expected logarithm of the failure times as

$$\ln(t_i) = \frac{Y_i}{\beta} + \ln(\eta). \tag{34}$$

Step 5 From the $\ln(t_i)$ data of step 4, estimate μ_x by using Equation 2, and σ_x by using Equation 1. Then from the $\ln(t_i)$ data, select the *USL* as the maximum $\ln(t_i)$ value [$USL = \ln(t_{max})$] and the *LSL* as the minimum $\ln(t_i)$ value [$LSL = \ln(t_{min})$]. Observe that μ_y and σ_y also can be estimated by using Equations 28 and 30.

Step 6 By using data of step 5 into Equations 31 and 32, estimate the corresponding C_p and C_{pl} indices.

TABLE 1 Weibull data for $R(t)$ = 0.96, t = 1500 h, $\beta = 3$, and η = 4356.388 h

n	Y_i	$ln(t_i)$	t_i	n	Y_i	$ln(t_i)$	t_i
1	−3.557180	7.193672	1330.98	15	−0.113530	8.341555	4194.61
2	−2.648949	7.496416	1801.57	16	−0.004111	8.378028	4350.42
3	−2.164632	7.657854	2117.21	17	0.105271	8.414489	4511.97
4	−1.827031	7.770388	2369.39	18	0.216016	8.451404	4681.64
5	−1.564383	7.857937	2586.18	19	0.329829	8.489342	4862.66
6	−1.347081	7.930371	2780.46	20	0.448989	8.529061	5059.69
7	−1.159967	7.992743	2959.40	21	0.576870	8.571688	5280.04
8	−0.994199	8.047999	3127.53	22	0.719129	8.619108	5536.45
9	−0.844133	8.098021	3287.95	23	0.886994	8.675063	5855.07
10	−0.705909	8.144095	3442.99	24	1.110293	8.749496	6307.51
11	−0.576736	8.187153	3594.47	24.496598	1.272959	8.803718	6658.96
12	−0.454489	8.227902	3743.97	$\mu_y = -0.5143077$		$\mu_x = 8.2079626$	
13	−0.337469	8.266909	3892.90	$\sigma_y = 1.1983030$		$\sigma_x = 0.3994343$	
14	−0.224243	8.304651	4042.63	$Cp = 0.671803$		$Cpk = 0.846439$	

5.2 | Application

Step 1 Suppose, the customer requirements are $R(t) = 0.96$, $t = 1500$ hours, and from line base product, $\beta = 3$. Thus, from Equation 15, $\eta = 4356.388$ hours

Step 2 The Weibull family is W(3, 4356.388).

Step 3 From Equation 16, $n = 24.4966$parts; from Equation 2, $\mu_y = -0.5143077$; and from Equation 27, $\sigma_y = 1.1983030$. The Y elements are given in Table 1.

Step 4 The logarithm of the lifetime data is given in Table 1.

Step 5 From Equation 2, $\mu_x = 8.2079626$. From Equation 1, $\sigma_x = 0.3994343$. And from data of step 4, $USL = \ln(t_{\max}) = 8.8037183$, $LSL = \ln(t_{\min}) = 7.1936717$.

Step 6 From Equation 31, the $Cp = 0.671803$ and from Equation 32, the $Cpk = 0.846439$.

On the other hand, by using Equations 9 to 12, we observe that from Equation 9, $\mu_x = 8.186993$, and from Equation 10, $\sigma_x = 0.427517$. Thus, from Equation 11, $Cp = 0.627674$ and from Equation 12, $Cpk = 0.77449$, which clearly are significantly different from those estimated in step 6. Therefore, because $\gamma > \mu_y$ (0.577216 > 0.5143077) and $\sqrt{6}/\pi > \sigma_y$ (1.28255 > 1.1983030), then by using Equation 9, μ_x was subestimated and by using Equation 10 σ_x was overestimated, and as a consequence, the Cp and the Cpk indices were subestimated also. Finally, it is important to mention that in contrast to Equations 11 and 12, the indices defined in Equations 31 and 32 are now sensitive to the sample size n, and thus, they represent the analyzed process.

As an additional comment, observe that because in the analysis, both the lognormal and the Weibull distribution use the logarithm data, then the PCIs defined in Equations 31 and 32 also works in the lognormal case. The only difference between both distributions is how the UCL and LCL values are estimated. And because in both cases the UCL and LCL values are determined from the maximal and minimal expected failure time values, then the difference between the UCL and LCL of both distributions is only due to the difference between their maximum and minimum expected values.

6 | CONCLUSIONS

Clearly, Equations 31 and 32 can be used to estimate the PCIs for either short or long term. The short or long term depends if μ_x and σ_x represent the process behavior for long or short term. The proposed PCIs, by incorporating the effect of n over μ_x and σ_x, automatically correct the bias generated on them by the use of $\gamma > \mu_y$ in Equation 9 and by the use of $\sqrt{6}/\pi > \sigma_y$ in Equation 10. Even more, since the proposed method completely determine the PCIs for any β value, or operational environment, which could be $\beta < 1$, $\beta = 1$ or $\beta > 1$ (see Rine[22] Section 2.3), then the proposed method can also be used to estimate the PCIs from any accelerated lifetime analysis. On the other hand, note that the estimated PCIs in Equations 31 and 32 are unique for the desired $R(t)$ index, thus, to increase them, μ_x has to be increased and/or σ_x has to be reduced, but this implies that $R(t)$ also increases. Therefore, for a specific $R(t)$ index, the PCIs are unique and independent on the used t, β, and η values. On the other hand, since the estimated μ_x

and σ_x values completely determine η and β, and because μ_x and σ_x directly represents the analyzed process, then μ_x and σ_x should be used to monitoring the process as it is proposed in Piña.[23] Furthermore, since the relations given in Equations 28 and 30 can be used to estimate the Gumbel parameters in function on n, then Equations 28 and 30 can replace Equations 9 and 10 in any Gumbel analysis.

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Jesús F. Ortiz-Yañez works as reliability engineer in the validation department at Ted de México S.A de C.V. He completed his PhD in Science in Engineering on 2016 at the Universidad Autónoma de Ciudad Juárez. He focuses his research on Weibull and lognormal reliability analysis.

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